AIMS Biophysics, 9(1): 61–71.
DOI: 10.3934/biophy.2022006
Received: 20 January 2022
Revised: 27 February 2022
Accepted: 07 March 2022
Published: 18 March 2022

AIMS Biophysics

http://www.aimspress.com/journal/biophysics

Research article

Density of electric field energy around two surface-charged spheres surrounded by electrolyte II. The smaller sphere is inside the larger one

István P. Sugár*

Department of Neurology, Icahn School of Medicine at Mount Sinai, New York, NY 10029

* **Correspondence:** Email: istvansugar0@gmail.com.

Appendix 1

The Poisson equation is [8]:

$$\nabla^2 V(\underline{r}) = -\frac{\rho(\underline{r})}{\varepsilon_0 \varepsilon_r} \quad \text{(A1)}$$

where $\rho(\underline{r})$ is the charge density, ε_0 is the vaacum permittivity and ε_r is the relative static permittivity.
The solution of the above Poisson equation is [8]:

$$V(\underline{r}) = \iiint d^3\underline{r}' \frac{\rho(\underline{r}')}{4\pi\varepsilon_0\varepsilon_r|\underline{r}-\underline{r}'|} \quad \text{(A2)}$$

In the case of the Poisson-Boltzmann equation [8, 9] $-\frac{\rho(\underline{r})}{\varepsilon_0\varepsilon_r}$ (in Eq A1) is equal with

$$-\frac{\rho(\underline{r})}{\varepsilon_0\varepsilon} = \sum_i \frac{z_i q n_i^0}{\varepsilon_0\varepsilon_r} e^{-\frac{z_i q V(\underline{r})}{k_B T}} \quad \text{(A3)}$$

where q is the elementary charge (positive or negative depending on the charge of the i-th type of ion), z_i is the charge number of the i-th type of ion, k_B is the Boltzmann constant, T is the absolute temperature, $\rho(\underline{r})$ is the charge density of the ions in the electrolyte and ε_r is the relative static permittivity of the electrolyte.
The Screened Poisson equation is [8]:

$$\nabla^2 V(\underline{r}) - \lambda_D^{-2} V(\underline{r}) = -\frac{\rho_{ex}(\underline{r})}{\varepsilon_0 \varepsilon_r} \tag{A4}$$

where $\rho_{ex}(\underline{r})$ is the density of the external charge at position $\underline{r}$, ε_0 is the electric constant and ε_r is the relative static permittivity of the electrolyte, λ_D is the Debye length. Note, that Eq A4 is valid if the electrolyte itself is electrically neutral. The solution of this equation is [8]:

$$V(\underline{r}) = \iiint d^3\underline{r}' \frac{\rho_{ex}(\underline{r}')}{4\pi\varepsilon_0\varepsilon_r} \frac{e^{-|\underline{r}-\underline{r}'|/\lambda_D}}{|\underline{r}-\underline{r}'|} \tag{A5}$$

i.e. the potential is the superposition of the so called screened Coulomb potential of the external charges.

Appendix 2 The electric field energy density of N+1 charged spheres surrounded by electrolyte where N small spheres located within a large sphere

The coordinates of the center of the larger sphere are $(x_L, y_L) = (0, 0)$

$$u_F(x_P, y_P, z_P) = \frac{\varepsilon_r\varepsilon_0}{2}\underline{E}\cdot\underline{E} = \frac{\varepsilon_r\varepsilon_0}{2}\left(\{E_{Lx} + \sum_{i=1}^{N} E_{ix}\}^2 + \{E_{Ly} + \sum_{i=1}^{N} E_{iy}\}^2 + \{E_{Lz} + \sum_{i=1}^{N} E_{iz}\}^2\right) \tag{A6}$$

where the electric field created by the i-th sphere at point P1 is:

$$\underline{E_i} = (E_{ix}, E_{iy}, E_{iz}) = -\frac{dV_i}{dD_i}\left(\frac{x_P - x_i}{D_i}, \frac{y_P - y_i}{D_i}, \frac{z_P - z_i}{D_i}\right) \tag{A7}$$

where $D_i = \sqrt{(x_P - x_i)^2 + (y_P - y_i)^2 + (z_P - z_i)^2}$ is the distance between point P1 and the center of the i-th charged sphere and

$$\frac{dV_i}{dD_i} = \begin{cases} \frac{k_e Q_i \lambda_D}{\varepsilon_r R_i} sinh(R_i/\lambda_D)\left[-\frac{e^{-\frac{D_i}{\lambda_D}}}{D_i^2} - \frac{e^{-\frac{D_i}{\lambda_D}}}{D_i\lambda_D}\right] & if\ D_i > R_i \\ \frac{k_e Q_i \lambda_D}{\varepsilon_r R_i} e^{-\frac{R_i}{\lambda_D}}\left[-\frac{sinh\left(\frac{D_i}{\lambda_D}\right)}{D_i^2} + \frac{cosh\left(\frac{D_i}{\lambda_D}\right)}{D_i\lambda_D}\right] & if\ D_i < R_i \end{cases}$$

where $0 < \sqrt{x_i^2 + y_i^2} < R_L - R_i$ and $\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2} > R_i + R_j$ where $i \neq j$. Q_i is the total surface charge of the i-th small sphere of radius R_i.