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Research article

Dynamical analysis of fractional-order chemostat model

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Appendix

Algorithm of Adams-type Predictor Corrector Method:

$h := \dfrac{T}{N}$

$m := \lceil \alpha \rceil$

for $k = 1$ to N do begin

$b_k := k^{\alpha} - (k-1)^{\alpha}$

$a_k := (k+1)^{\alpha+1} - 2k^{\alpha+1} + (k-1)^{\alpha+1}$

end

$JJ[0] := JJ_0[0]$

for $j = 1$ to N do begin

$$pJ := \sum_{k=0}^{m-1} \frac{(jh)^k}{k!} JJ_0(k) + \frac{h^{\alpha}}{\Gamma(\alpha+1)} \sum_{k=0}^{j-1} b(j-k)\, fJ(kh, JJ(k))$$

$$JJ(k) := \sum_{k=0}^{m-1} \frac{(jh)^k}{k!} JJ_0(k) + \frac{h^{\alpha}}{\Gamma(\alpha+2)} \left[\left((j-1)^{\alpha+1} - (j-1-\alpha) j^{\alpha}\right) fJ(0, JJ(0))\right]$$
$$+ \frac{h^{\alpha}}{\Gamma(\alpha+2)} \left[fJ(jh, pJ) + \sum_{k=1}^{j-1} a(j-k)\, fJ(kh, JJ(k)) \right]$$

end

Proof of Proposition 1: The two roots of the characteristic polynomial are expressed as,

$$\lambda_{\pm} = -\frac{-b \pm \sqrt{b^2 - 4c}}{2}.$$

Equation (2.4) is equivalent to the Routh-Hurwitz case if both roots are real or complex conjugate with negative real parts. Otherwise, the roots become

$$\lambda_{\pm} = -\frac{-b \pm i\sqrt{4c - b^2}}{2},$$

and get Equation (2.6) if both roots are complex conjugate with positive real parts.
The mathematical explanation of Routh-Hurwitz condition: By substituting values from Table 1, the eigenvalues can be obtained as

$$P(\lambda) = \lambda^2 + b\lambda + c,$$

$$P(\lambda) = \lambda^2 + \frac{2552}{199875}\lambda + \frac{49}{18750},$$

$$b = \frac{2552}{199875} > 0, \qquad c = \frac{49}{18750} > 0.$$

This show that the condition in Eq. (2.5) is satisfied which is $b > 0$ and $c > 0$.