



Research article

A new mathematical modelling and parameter estimation of COVID-19: a case study in Iraq

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Abstract: Mathematical modelling has been widely used in many fields, especially in recent years. The applications of mathematical modelling in infectious diseases have shown that situations such as isolation, quarantine, vaccination and treatment are often necessary to eliminate most infectious diseases. In this study, a mathematical model of COVID-19 disease involving susceptible (S), exposed (E), infected (I), quarantined (Q), vaccinated (V) and recovered (R) populations is considered. In order to show the biological significance of the system, the non-negative solution region and the boundedness of the relevant biological compartments are shown. The endemic and disease-free equilibrium points of the model are calculated, and local stability analyses of these equilibrium points are performed. The basic reproduction number is also calculated for the relevant model. Sensitivity analysis of this number is studied, and it has been pointed out which parameters affect this number and how they affect it. Moreover, using real data from Iraq, the model parameters are estimated using the least squares curve fitting method, and numerical simulations are performed by using these estimated values. For the solution of the model, the Adams-Bashforth type predictive-corrective numerical method is used, and with the help of numerical simulations, several predictions are achieved about the future course of COVID-19.

Keywords: Basic reproduction number; COVID-19; numerical simulation; parameter estimation; stability analysis; sensitivity analysis

Appendix A.

A.1. Adams-Bashforth method

Algorithms and numerical solutions for the *SEIQVR* model using Multi-Step Methods are discussed. The last two methods used to generate numerical solutions are the Runge-Kutta and Adams-Bashforth methods. The predictor-corrector method involves using the Adams-Bashforth, Runge-Kutta as well as Adams-Moulton methods for more accurate results. In this study, we consider the four-step Adams-Bashforth method.

The Adams-Bashforth four-step method is an open method that finds w_{i+1} using four steps [39]. The method is defined as:

$$w_0 = \alpha, w_1 = \alpha_1, w_2 = \alpha_2, w_3 = \alpha_3,$$

$$w_{i+1} = w_i + \frac{h}{24} (55f(t_i, w_i) - 59f(t_{i-1}, w_{i-1}) + 37f(t_{i-2}, w_{i-2}) - 9f(t_{i-3}, w_{i-3})).$$

A.2. Four-step Adams-Bashforth method algorithm

In general, the algorithm for the four-step Runge-Kutta method and the Adams-Bashforth method is as follows:

INPUT: endpoints t_{\min} , t_{\max} start and end points, respectively; N is an integer; α is the initial condition

OUTPUT: approximations w to y at the $(N+1)$ values of t

- Step 1: Set

$$h = \frac{t_{\max} - t_{\min}}{N}, t_0 = t_{\min}, w_0 = \alpha.$$

- Step 2: For $i = 1, 2, 3$ do and repeat Steps 3, 4, and 5.
- Step 3: Set

$$F_1 = hf(t_{i-1}, w_{i-1}),$$

$$F_2 = hf\left(t_{i-1} + \frac{h}{2}, w_{i-1} + \frac{F_1}{2}\right),$$

$$F_3 = hf\left(t_{i-1} + \frac{h}{2}, w_{i-1} + \frac{F_2}{2}\right),$$

$$F_4 = hf(t_{i-1} + h, w_{i-1} + F_3),$$

$$w_i = w_{i-1} + \frac{1}{6}(F_1 + 2F_2 + 2F_3 + F_4),$$

$$t_i = t_{\min} + ih.$$

- Step 4: OUTPUT(t_i, w_i).
- Step 5: For $i = 4, \dots, N$, do Steps 6–9.

- Step 6: Set

$$w_i = w_3 + \frac{h}{24} [55f(t_3, w_3) - 59f(t_2, w_2) + 37f(t_1, w_1) - 9f(t_0, w_0)],$$

$$t_i = t_{\min} + ih.$$

- Step 7: OUTPUT(t_i, w_i)
- Step 8: For $j = 0, 1, 2$, set

$$t_j = t_{j+1}, \quad w_j = w_{j+1}.$$

- Step 9: Set
(Preparation for the next iteration)

$$t_3 = t, \quad w_3 = w.$$

- Step 10: STOP

We have the following algorithm for our *SEIQVR* model that we used in the study:

- Step 1: For the systems of equations set the initial conditions and parameters:

$$\beta_1, \eta, k_1, \varepsilon_1, \varepsilon_2, \theta_1, \theta_2, \mu, \mu_1, S(0) = S_0, E(0) = E_0, I(0) = I_0, Q(0) = Q_0, V(0) = V_0,$$

$$R(0) = R_0$$

- Step 2: For $i = 1, 2, 3$, do and repeat Steps 3 and 4.
- Step 3: Set

$$F_{S1} = hf_1(S_{i-1}, E_{i-1}, V_{i-1}),$$

$$F_{E1} = hf_2(S_{i-1}, E_{i-1}),$$

$$F_{I1} = hf_3(E_{i-1}, I_{i-1}),$$

$$F_{Q1} = hf_4(E_{i-1}, I_{i-1}, Q_{i-1}),$$

$$F_{V1} = hf_5(S_{i-1}, V_{i-1}),$$

$$F_{R1} = hf_6(I_{i-1}, Q_{i-1}, R_{i-1}),$$

and,

$$\begin{aligned}
F_{S_2} &= hf_1 \left(S_{i-1} + \frac{1}{2} F_{S_1}, E_{i-1} + \frac{1}{2} F_{E_1}, V_{i-1} + \frac{1}{2} F_{V_1} \right), \\
F_{E_2} &= hf_2 \left(S_{i-1} + \frac{1}{2} F_{S_1}, E_{i-1} + \frac{1}{2} F_{E_1} \right), \\
F_{I_2} &= hf_3 \left(E_{i-1} + \frac{1}{2} F_{E_1}, I_{i-1} + \frac{1}{2} F_{I_1} \right), \\
F_{Q_3} &= hf_4 \left(E_{i-1} + \frac{1}{2} F_{E_1}, I_{i-1} + \frac{1}{2} F_{I_1}, Q_{i-1} + \frac{1}{2} F_{Q_1} \right), \\
F_{V_1} &= hf_5 \left(S_{i-1} + \frac{1}{2} F_{S_1}, V_{i-1} + \frac{1}{2} F_{V_1} \right), \\
F_{R_1} &= hf_6 \left(I_{i-1} + \frac{1}{2} F_{I_1}, Q_{i-1} + \frac{1}{2} F_{Q_1}, R_{i-1} + \frac{1}{2} F_{R_1} \right), \\
F_{S_3} &= hf_1 \left(S_{i-1} + \frac{1}{2} F_{S_2}, E_{i-1} + \frac{1}{2} F_{E_2}, V_{i-1} + \frac{1}{2} F_{V_2} \right), \\
F_{E_3} &= hf_2 \left(S_{i-1} + \frac{1}{2} F_{S_2}, E_{i-1} + \frac{1}{2} F_{E_2} \right), \\
F_{I_3} &= hf_3 \left(E_{i-1} + \frac{1}{2} F_{E_2}, I_{i-1} + \frac{1}{2} F_{I_2} \right), \\
F_{Q_3} &= hf_4 \left(E_{i-1} + \frac{1}{2} F_{E_2}, I_{i-1} + \frac{1}{2} F_{I_2}, Q_{i-1} + \frac{1}{2} F_{Q_2} \right), \\
F_{V_3} &= hf_5 \left(S_{i-1} + \frac{1}{2} F_{S_2}, V_{i-1} + \frac{1}{2} F_{V_2} \right), \\
F_{R_3} &= hf_6 \left(I_{i-1} + \frac{1}{2} F_{I_2}, Q_{i-1} + \frac{1}{2} F_{Q_2}, R_{i-1} + \frac{1}{2} F_{R_2} \right),
\end{aligned}$$

and finally,

$$\begin{aligned}
F_{S_4} &= hf_1 (S_{i-1} + F_{S_3}, E_{i-1} + F_{E_3}, V_{i-1} + F_{V_3}), \\
F_{E_4} &= hf_2 (S_{i-1} + F_{S_3}, E_{i-1} + F_{E_3}), \\
F_{I_4} &= hf_3 (E_{i-1} + F_{E_3}, I_{i-1} + F_{I_3}), \\
F_{Q_4} &= hf_4 (E_{i-1} + F_{E_3}, I_{i-1} + F_{I_3}, Q_{i-1} + F_{Q_3}), \\
F_{V_4} &= hf_5 (S_{i-1} + F_{S_3}, V_{i-1} + F_{V_3}), \\
F_{R_4} &= hf_6 (I_{i-1} + F_{I_3}, Q_{i-1} + F_{Q_3}, R_{i-1} + F_{R_3}).
\end{aligned}$$

Then,

$$\begin{aligned}
S_i &= S_{i-1} + \frac{1}{6}(F_{S1} + 2F_{S2} + 2F_{S3} + F_{S4}), \\
E_i &= E_{i-1} + \frac{1}{6}(F_{E1} + 2F_{E2} + 2F_{E3} + F_{E4}), \\
I_i &= I_{i-1} + \frac{1}{6}(F_{I1} + 2F_{I2} + 2F_{I3} + F_{I4}), \\
Q_i &= Q_{i-1} + \frac{1}{6}(F_{Q1} + 2F_{Q2} + 2F_{Q3} + F_{Q4}), \\
V_i &= V_{i-1} + \frac{1}{6}(F_{V1} + 2F_{V2} + 2F_{V3} + F_{V4}), \\
R_i &= R_{i-1} + \frac{1}{6}(F_{R1} + 2F_{R2} + 2F_{R3} + F_{R4}).
\end{aligned}$$

- Step 4: OUTPUT $(t_i, S_i, E_i, I_i, Q_i, V_i, R_i)$.
- Step 5: For $i = 4, \dots, N$, do Steps 6 and 7.
- Step 6: Set

$$\begin{aligned}
t_i &= t_{\min} + ih, \\
S &= S_3 + \frac{h}{24} [55f_1(S_3, E_3, V_3) - 59f_1(S_2, E_2, V_2) + 37f_1(S_1, E_1, V_1) - 9f_1(S_0, E_0, V_0)], \\
E &= E_3 + \frac{h}{24} [55f_2(S_3, E_3) - 59f_2(S_2, E_2) + 37f_2(S_1, E_1) - 9f_2(S_0, E_0)], \\
I &= I_3 + \frac{h}{24} [55f_3(E_3, I_3) - 59f_3(E_2, I_2) + 37f_3(E_1, I_1) - 9f_3(E_0, I_0)], \\
Q &= Q_3 + \frac{h}{24} [55f_4(E_3, I_3, Q_3) - 59f_4(E_2, I_2, Q_2) + 37f_4(E_1, I_1, Q_1) - 9f_4(E_0, I_0, Q_0)], \\
V &= V_3 + \frac{h}{24} [55f_5(S_3, V_3) - 59f_5(S_2, V_2) + 37f_5(S_1, V_1) - 9f_5(S_0, V_0)], \\
R &= R_3 + \frac{h}{24} [55f_6(I_3, Q_3, R_3) - 59f_6(I_2, Q_2, R_2) + 37f_6(I_1, Q_1, R_1) - 9f_6(I_0, Q_0, R_0)].
\end{aligned}$$

- Step 7: OUTPUT (t, S, E, I, Q, V, R) .
- Step 8: For $j = 0, 1, 2$, set

$$\begin{aligned}
t_j &= t_{j+1}; \\
S_j &= S_{j+1}, \\
E_j &= E_{j+1}, \\
I_j &= I_{j+1}, \\
Q_j &= Q_{j+1}, \\
V_j &= V_{j+1}, \\
R_j &= R_{j+1}.
\end{aligned}$$

• Step 9: Set

$$\begin{aligned}t_3 &= t, \\S_3 &= S, \\E_3 &= E, \\I_3 &= I, \\Q_3 &= Q, \\V_3 &= V, \\R_3 &= R.\end{aligned}$$

• Step 10: STOP.



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